RAN-2103000206020036

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T.Y.B.Sc. (Sem. VI) Examination March - 2025

Mathematics : MTH - 606

Number Theory - II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc. (Sem. VI)**

Name of the Subject :
☛ **Mathematics : MTH - 606 Number Theory - II**

Subject Code No.: **2103000206020036**

Seat No.:

Student's Signature

Q-1: **Answer Five any of the following:** **(10)**

(1) Write the condition that the linear congruence $ax = b \pmod{n}$ has a solution.

(2) If gcd $(a, 35) = 1$, show that $a^{12} = 1 \pmod{35}$.

(3) Find the all forms of positive integers satisfying $\tau(n) = 15$, what is the smallest positive integer for which this is true.

(4) If n is an even integer then prove that $\phi(2n) = 2\phi(n)$.

(5) Find a solution of the linear congruence $5x = 2 \pmod{26}$.

(6) Without using Wilson's theorem, verify that $(p-1)! = -1 \pmod{p}$ where $p = 7$.

(7) For $k \geq 2$, show that $n = 2^{K-1}$ satisfies $\sigma(n) = 2n - 1$.

(8) Deduce the Fermat's theorem from Euler's theorem.

Q-2: **Answer any two questions:** **(10)**

1) Let $n_1, n_2, \ldots, n_r$ be positive integers such that gcd $(n_i, n_j) = 1$ for $i \neq j$. Then the system of linear congruence

$x = a_1 \pmod{n_1}$

$x = a_2 \pmod{n_2}$

$\vdots$

$x = a_r \pmod{n_r}$

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has a simultaneous solution, which is unique modulo the integer $n_1.n_2 \dots n_r$.

(2) Solve the linear congruence $9x = 21 \pmod{30}$.

(3) Obtain the three consecutive integers each having square factors.

Q-3: Answer any two questions: (10)

(1) Define Pseudo Prime. Prove that if n is odd pseudo prime then $M_n = 2^n - 1$ is a larger one.

(2) (i) For prime p show that $1^{p-1} + 2^{p-1} + \dots + (p-1)^{(p-1)} = -1 \pmod{p}$.

(ii) For odd prime p show that $1^p + 2^p + \dots + (p-1)^p = 0 \pmod{p}$.

(3) If p is a prime then $(p-1)! = -1 \pmod{p}$.

Q-4: Answer any two questions: (10)

(1) Find the highest power of 10 which divides 100!.

(2) Prove that Mobius function is multiplicative.

(3) If $2^k - 3$ is prime then show that $n = 2^{k-1}(2^k - 3)$ satisfies $\sigma(n) = 2n + 2$.

Q-5: Answer any two questions: (10)

(1) If P is a prime and $k \geq 2$ then show that $\phi\left(\phi(p^k)\right) = p^{k-2}\,\phi\left((p-1)^2\right)$.

(2) If the integer $n > 1$ has r distinct odd prime factors then prove that $2^r \mid \phi(n)$.

(3) Using Euler's theorem to find the last two digits in the decimal expansion of 3^{253}.